C12 Decoupling of Lagrangian Equations of Motion to Improve Computational Efficiency and Application to Multi-Body Constrained Biomedical Engineering Systems

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The goal of this presentation is to develop a unique method for formulation of equations of motion based on Lagrangian energy methods that result in decoupling of the equations of motion in the highest order terms. This derivation yields equations of motion that are in a more convenient and efficient form for numerical analysis.

This presentation will impact the forensic community and/or humanity by demonstrating a method which reduces computer processing time by 10-17% which is significant when utilizing a high capacity computer to analyze occupant motion. Computer processing costs are also reduced proportionally.

As simulation of human occupant kinematics becomes increasingly complex, computer processing effectiveness becomes an increasingly important consideration. Decoupling the highest order terms improves the efficiency of numerical analysis, especially for nonlinear systems. More efficient processing yields results in less computer time or with less computer processing capability. This method therefore, may widen the practical use of occupant kinematic analytical tools.

Methodologically, the only work on decoupling equations of motion by Loduha and Ravani (1995) utilized Kane's method. Here, a method for decoupling the equations of motion is derived for the first time for use with Lagrangian energy based methods, commonly used in BioMedical Engineering. Methodology involves mathematical transformation of the equations of motion. Specifically, the equations of motion are decoupled in the highest order via a congruent transformation to a diagonal matrix called the *canonical form of a symmetric matrix under congruent transformations.* This was first attempted by Wade in 1951; here, congruent transformation will be utilized for the first time to decouple Lagrangian equations.

Highest order decoupling of Lagrangian equations of motion for constrained multi-rigid body systems is formulated to create equations of motion that are decoupled in the highest order terms. The practical benefit is less computer processing time, which is most apparent for complex systems. Decoupling the equations of motion in the highest order derivative terms results in a more convenient form of the equations for numerical analysis. Specifically, if the equations of motion are decoupled in the highest order prior to integration, computer processing is more efficient (especially for nonlinear systems) as decoupling of the highest order is no longer required at each step.

Mathematical derivation requires seven steps:

1. Coordinate Selection
2. Lagrangian, Kinetic Coenergy, and Potential Energy Transformations
3. Partial Derivative Transformations
4. Transformed Lagrangian Derivation
5. Transformation of Kinetic Coenergy
6. Transformation of Potential Energy
7. Selection of the Rate Transformation Matrix

In conclusion, a parsimonious side impact occupant kinematics example demonstrates that the selection of generalized velocity components as a function of generalized coordinates indeed results in decoupling of the Lagrangian equations of motion in the highest order. Further, a simplified BioMedical Model shows that the decoupled equations and the coupled equations of motion yield the same result. The difference is the decoupled equations a require 10-17% less numerical processing time.

Mathematical Modeling, Simulation, Human Kinematics

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