

## C4 Vehicle Stability

Richard W. McLay, PhD\*, STARK rpx, 1231 Hamilton Court, Iowa City, IA 52245; and Donald J. Anderson, BSME, Anderson Engineers, 13176 Pierce Street, NE, Blaine, MN 55434

After attending this presentation, attendees will gain the ability to compare and make use of two analyses for vehicle stability: 1) a one degree-of-freedom analysis; and 2) a five-degree-of-freedom analysis for studying the point at which a vehicle becomes unstable in a turn.

This presentation will impact the forensic community and/or humanity by providing methods to ensure fairness in the presentation of vehicle stability evidence for both criminal and civil actions.

Vehicle stability for a real, four-wheeled car is extremely complex. The basic model for that vehicle type is shown in FIG. 1, with examples of some of the applied forces.

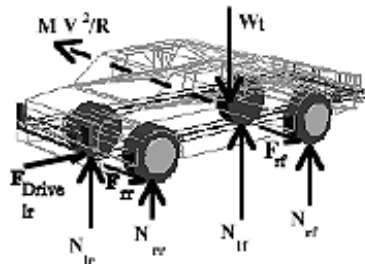


FIG. 1: Model for vehicle stability

Note that the tires are subject to normal forces, side friction forces, and driving or braking friction forces, which obey the inequality for the friction circle at each wheel. Five degrees-of-freedom describe this model: vertical and lateral deflections with yaw, pitch, and roll rotations, illustrated in FIG. 2.

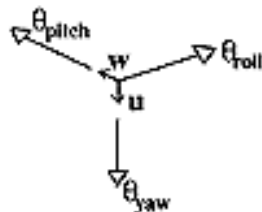


FIG. 2: Model degrees-of-freedom

The model contains two types of structures that store elastic energy: the suspension at each wheel and the lateral spring for each tire. The forces on the car in a turn cause it to pitch forward against the suspension, to roll laterally against the suspension, and to displace and yaw laterally against the lateral stiffness of the tires. All five degrees-of-freedom are required to analyze the stability of the four-wheeled vehicle. Apart from the complexities of the tire behavior when a steer angle is introduced, what makes this problem difficult is the fact that it is three times statically indeterminate: researchers can remove one wheel and theoretically the car will stand on three supports. One front and one rear wheel on ice and the car will again theoretically be supported in a turn by only two tires that deflect laterally. Because the problem is statically indeterminate, the analysis was carried out by using elasticity theory, specifically the theorem of minimum potential energy (PE) with a form of the finite element method. For roll, pitch and vertical deflection, the linear problem uncouples with a potential energy:

$$PE = 1/2 [ K_{front}(u+a q_{pitch}+TW q_{roll})^2+K_{front}(u+q q_{pitch}-TW q_{roll})^2 +K_{rear}(u-b q_{pitch}+TW q_{roll})^2+K_{rear}(u-b q_{pitch}-TW q_{roll})^2 ] - Wt u - h q_{roll} (Wt V^2/gR_1) \cos q_{cg} - h q_{pitch} (Wt V^2/gR_1) \sin q_{cg}$$

Four examples were studied using this theory and basic experiments were run using a tire fixture with various inflation pressures on a standard asphalt surface. The conclusion of the paper is that this model for vehicle stability, which examines the condition at each wheel, shows the true critical point at which the vehicle becomes unstable. This is in contrast to the simplified one- degree-of-freedom equation result that: 1) ignores the



## Engineering Section – 2006

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friction circle effects at the wheels, 2) assumes that the vehicle is always in a neutral steer condition, and 3) places the center-of-gravity on the pavement such that the effects of suspension in roll are negligible.

Some specific results for the example problem comparing this analysis with a one-degree-of-freedom (ODF) result gives the following:

$$R = 862. \text{ feet } m = 0.68$$

$$V_{\text{ODF}} = \sqrt{m G R} = 138 \text{ ft/sec (94 mph)} \quad V_{\text{Model}} = 84 \text{ ft/sec (57 mph)}$$

At that speed, the four-wheel vehicle model shows the vehicle deflects its suspension 8.8 inches on average, it pitches forward  $5^\circ$ , the roll angle, to the outside of the turn, is  $3^\circ$ , the lateral tire displacement is 1.9 inches, and the vehicle rotates in yaw against the tire stiffness an angle of  $0.5^\circ$ . The normal force on the right-rear tire is  $n_r = 177$  lbs, while the lateral force required for equilibrium is  $F_{arr} = 124$  lbs. This is an under steer vehicle and would be expected to become unstable and yaw clockwise, breaking traction on the right-rear tire in a turn to the right. In contrast, the one-degree-of-freedom model requires that all four wheels be in the critical condition, which is not true for this nose-heavy under steer vehicle.

### **Stability, Yaw, Potential Energy**