C23 Converging Evidence: A Bayesian Example

Chesterene L. Cwiklik, BS, Pacific Coast Forensic Science Institute, Inc. and Cwiklik & Associates, 2400 6th Avenue South #256 and #257, Seattle, WA 98134; and Andrew J. Friedman, BS, BA, OGW Research, Inc., 4253 Woodland Park Avenue North, Seattle, WA 98103*

After attending this presentation, attendees will learn how mutually exclusive hypotheses and Bayesian inference are practical tools that can help sort and evaluate analytical information in complex cases. This presentation will incorporate examples from environmental casework.

This presentation will impact the forensic community and/or humanity by demonstrating the use of mutually exclusive hypotheses and Bayesian inference can assist forensic practitioners in meeting the interests of the courts in presenting defensible and falsifiable analyses in environmental and other cases. It should aid practitioners in performing focused work that can address the issues of interest in a scientifically reliable way.

Preliminary results for a contaminated site performed to determine regulatory compliance do not always answer questions about who is responsible and what they should do about it. As a case moves from regulatory compliance to forensic investigation to litigation to remediation, and the first rounds of testing do not yield a simple and obvious answer, it is useful to have a few tools to sort through the information and to evaluate what the results mean. This is especially important when things are complicated and multiple hypotheses can be offered in explanation. A clean experiment means that the scientist or engineer knows what questions have and have not been answered. It is critical to focus the analytical questions so that any additional work would have an impact on the case questions, and to eliminate untenable hypotheses so the remaining ones can be evaluated. The focus of this paper is on hypothesis formation after some of the results are in, and the use of formal mutually exclusive hypotheses and Bayesian inference as a means for narrowing down hypotheses, focusing any testing that should still be done, and weighing the final hypotheses.

For mutually exclusive hypotheses A and B, if A is true, B is not true, and if B is true, A is not true. A formal hypothesis suitable for comparing with its antithesis should be clean and simple, and any conditionals (the “true ifs”) placed into separate hypotheses so a comparison of hypotheses is easy to interpret. For example, if lead is found in soil samples from a site, and white paint chips and metal bearings are also found in the soil, the latter are both possible sources of lead, and there may be unknown contributors as well. One could construct the following hypotheses: 1a) the lead in the soil is from white paint chips and metal bearings only; or 1b) the lead in the soil is from white paint chips, metal bearings, and an unknown source. “A” would be true if no lead is found in the extracts. It may also be true if lead from either or both sources has leached into the soil. “B” would be true if lead is found in the extracts but in a form not explained by leaching of the paint or metal bearings. Lead in the extracts may also be from both leaching and another source. This does not provide a clean answer, so additional hypotheses may be needed: 2a) The lead in the soil sample extracts is that leached from white paint chips and metal bearings only; 2b) The lead in the soil sample extracts is from another source only; or 2c) the lead in the soil sample extracts is from both paint-and bearing- leached lead and from another source. These hypotheses can probably be resolved through additional testing, and the construction of the problem is now clean enough to allow focused testing. It may be that existing data can resolve them; the comparison of hypotheses allows for a quick determination of the needed data.

It may not be possible to test this further. When additional testing is not possible, the existing information must be evaluated and weighed. In comparing Hypotheses 2b and 2c, for example, the scientist or engineer should weigh the two possibilities to determine whether Hypothesis 2C is not only possible, but whether it is reasonably likely. It is also important to remember that something may be true even if is unlikely. That is why formal statistical evaluation or other evaluations of likelihood should not be attempted until testing to distinguish hypotheses has been performed. Other tools to use in evaluating data after additional testing is not possible include asking: “if it is not what I think it is, what else might it be?” This can produce additional hypotheses to evaluate and can help re-focus on the actual data. Another tool is to find the best fit with the evidence via evaluation of the convergence of data. A specific test result and analytical conclusion may have several possible explanations, but when all the test results and conclusions are considered, each of them may include one or two of the many possible data point explanations. This is where the data converges. The explanation where all the data converges is usually the best explanation, and an explanation where some of the data converges and none contradicts it is another possible explanation. The latter should be included in a reporting of results, as it may be true.

After the mutually exclusive hypotheses have been constructed, and any additional testing performed, the remaining hypotheses should be evaluated and weighed. A useful statistical tool for doing so is Bayesian inference. Bayes’s Rule expresses the probability that a certain event has occurred given a specific condition or conditions of measurement. It does this by relating the probability of the event given the evidence, to the probability of the evidence given the event. This is a way to measure the impact of the evidence on the overall probability. For example, it can be used to derive the probability that lead contamination is attributable to the peeling white paint of oil storage company tanks at the site being investigated, and metal bearings from the

 ** Presenting Author*

railroad previously on the site, given the condition that lead was found in the soil in the corresponding white paint chips, the metal bearings and the extracts. As a model, it is broadly applicable to evaluating scientific endeavors, and is of particular interest in comparing a hypothesis with an alternative hypothesis in light of a particular analytical result(s). In the aforementioned example, an alternative hypothesis might be that despite the presence of lead in the paint chips and metal bearings, the lead in the soil extracts is actually from another source.

A particularly useful form of Bayes's Rule for forensic scientists and engineers is to write the rule as a statement of odds rather than as a statement of probability, so that any probability is compared with its antithesis. This is what an odds statement entails. Thus, the odds of the event occurring given specific evidence are related to the overall odds of the event via a ratio. This ratio is called the Likelihood Ratio, i.e., a ratio of the probability of the evidence given the event, versus the probability of its being there even if something else happened instead. Thus, when two mutually exclusive hypotheses are being compared, the probability of one is being divided by the probability of other.

In mathematical language: Where J is the event (i.e., the scenario) and I is the condition (i.e., the evidence or result), Bayes's Rule — written in terms of odds rather than probability — is: $O(J|I) = O(J) [P(I|J)] / [P(I|\text{not-}J)]$, where $O(J|I)$ is the odds of event J occurring given the condition I; $O(J)$ is the odds of event J occurring; $P(I|J)$ is the probability of condition I being present if event J occurs; and $P(I|\text{not-}J)$ is the probability of condition I being present if event J does not occur

Even if a statistical evaluation cannot be performed for a lack of hard numbers, which may be difficult to come by when an unknown source is being considered, the reasoning involved in statistical evaluation can be applied, especially when Bayesian inference is used. It is more interesting to estimate some probabilities based upon date from other investigations to see how a comparison of the estimated probabilities would affect the overall questions being addressed. In this paper, the authors will present a Bayesian analysis of evidence in an environmental investigation using estimated probabilities of the data.

In summary, mutually exclusive hypotheses are a powerful tool in focusing analytical questions and narrowing down hypotheses in complex investigations once some of the analytical results have already come in. Bayesian statistical analysis is useful in evaluating and weighing the hypotheses that remain after testing has been completed. These tools are accessible to the forensic scientist and the forensic engineer, even those who do not have formal training in statistics, and are useful in a broad range of investigations.

Environmental Forensics, Forensic Science, Bayesian Inference

